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ABSTRACT

This Workshop Leader's Guide contains step-by-step procedures for preparing, organizing, and presenting 1-hour and 3-hour workshops on estimation in mathematics. It was designed to assist Technical Assistance Center staff members and other inservice providers in conducting successful workshops on estimation in mathematics for administrators, teachers, instructional assistants, and other associated with Chapter 1 mathematics programs. Included are an advance planner and a checklist of needed materials and equipment, the workshop outline, a review of literature on the topic, resource articles and support materials, and participant handouts and transparency masters. The bibliography contains 75 references. (KR)

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# ADVANCED SKILLS FOR CHAPTER 1 MATHEMATICS :

# ESTIMATION

## Workshop Leader's Guide

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# INTRODUCTION

## What is the Workshop Leader's Guide?

This Workshop Leader's Guide contains step-by-step procedures for preparing, organizing, and presenting one-hour and three-hour workshops on estimation in mathematics. It was designed to assist Technical Assistance Center staff members and other inservice providers in conducting successful workshops on estimation in mathematics for administrators, teachers, instructional assistants, and others associated with Chapter 1 mathematics programs.

## The Sections...

**Preliminaries** contains an Advance Planner and a detailed checklist of materials and equipment you will need to conduct a workshop.

The **Workshop Outline** describes specific procedures to follow for conducting a mathematics workshop on estimation for Chapter 1 staff. However, you may need to adapt and modify the suggested procedures to fit the needs of the specific workshop participants with whom you are working.

The following are included within the **Workshop Outline:**

- workshop goals
- workshop activities for increasing participants' background knowledge
- recommendations for using transparencies and handouts in workshop activities

The **Background** section provides a brief overview of research on estimation in mathematics to help inform you about the topic so you can present relevant, up-to-date information to workshop participants. For additional information, see the section on **Resource Articles** or the **Bibliography** for more references.

**Sections 2** and **3** contain the Participant Handout and Transparency Masters, respectively. **Section 4** contains the background paper. **Section 6** contains extra support materials that are not specifically referenced in the workshop but may be used as they suit your purposes.

## How to Use This Guide

This guide contains outlines for a one-hour and a three-hour workshop on estimation for Chapter 1 mathematics programs. The content of the workshop you decide to conduct will, of course, depend upon the amount of time, equipment, and space available, as well as the specific needs of participants.

**How to Use This Guide** (cont.)

The Guide contains the planning and presentation materials necessary to conduct a successful workshop on estimation in mathematics; however, these materials were developed to allow a great deal of flexibility. Suggestions for workshop variations and a variety of problems are included so that the workshop can be adjusted to suit your audience needs and mathematical background. A range of approaches to estimation instruction, types of problems, and specific research can be used in the actual workshop presentation. You may choose to change, add, or eliminate a problem, activity, or transparency. The sections in the Workshop Outline titled, "Participant Involvement with Estimation Strategies" and "Applying Estimation Strategies to Chapter 1 Instruction", are well suited for expansion or contraction, depending upon your presentation needs.

Just remember that the definitive word here is ***GUIDE***, both in terms of guiding you through all phases of the planning and presentation of the workshop, and in terms of allowing you many choices for personalizing your presentation. Using the Guide is simple, if you follow these steps:

- *Glance* through the entire set of materials. This will give you a feel for the types of materials contained in the Guide (and their location) when you study the details later.
- *Understand* as many of the materials contained in the Guide as possible. Plan enough time to develop a full grasp of the materials in order to make more informed decisions about your workshop presentation.
- *Investigate* further. You may want to do additional research, try different problems, or experiment with various activities.
- *Develop* additional materials. These may be workshop notes, transparencies, handout pages, activities, or any item resulting from your "investigating" activities.
- *Edit*. Look carefully at the total picture, then elaborate or eliminate, if necessary.

Remember that the key is to become familiar with the materials in the Guide, then to make adjustments to suit your needs and those of your audience.

## Getting Started

Once you have established some of the initial presentation details, such items as date, place, and type of audience, then you should begin studying the contents of the Workshop Leader's Guide. Follow the steps outlined in the section **How to Use This Guide--*Glance*, *Understand*, *Investigate*, *Develop*,** and ***Edit***. (The Advance Planner, a simple checklist for planning this or any workshop, is located in **Preliminaries.**)

Begin planning as soon as possible. Even if you use only the materials in the Guide, all of the steps--***G-U-I-D-E*** -- will take time and should be included in your planning. It is especially important to allow yourself the opportunity to thoroughly explore the activities of the workshop so the purpose and strategies for each problem will become more apparent to you. In addition, as you engage in the estimation workshop, you will likely discover that you need or want to try additional problems and activities from several sources. You may find them to be better suited to your specific purpose and prefer to substitute them for others in the Workshop.

The remaining sections of this guide include outlines for two workshops. Sections **2** and **3** contain masters for participant handouts and transparencies. Sections **4** and **5** include background information for both workshops. No matter what length workshop you plan to do, ***you will always start with the one-hour workshop.*** For example, the three-hour workshop includes the activities outlined for the one-hour workshop.

### The Workshop Experience

Encourage participants to think about their workshop experience in three ways:

√ as a *participant* experiencing the fun and satisfaction of using estimation.

√ as a *teacher* learning about the process of estimation, choosing good problems, and modeling problem-solving behavior.

√ as a *student* having the experience of using estimation. What do you think and feel? How do you approach, interpret, and understand the problem?

# Preliminaries

The masters for making handouts for participants are included in **Section 2**. Transparency masters are in **Section 3**. In **Section 6**, you will find additional support materials to use as you wish. Before the workshop, make the overhead projector transparencies you plan to use, and be sure you have one copy of the handouts for each participant.

If you are presenting the workshop at a new or unfamiliar location, ask your contact person to be sure that the equipment listed below is available and in working order on the scheduled day and time. If you will be supplying your own equipment, make arrangements for obtaining the equipment well in advance of the workshop. Be sure you have the following items before you begin.

**Equipment and Supplies:**

- ☐ Overhead Projector
  - extension cord
  - 3-way plug adaptor
  - extra bulb or spare projector
- ☐ Pens for Overhead Projector Transparencies
- ☐ Screen for Overhead Projector

**Materials:**

- ☐ Workshop Outline/Extra Notes
- ☐ Participant Handouts (one for each participant)
- ☐ Overhead Transparencies
- ☐ Blank Overhead Transparencies and Pens (at least three of each)

**Before you Begin**

- Be sure the Overhead Projector is properly focused.
- Be sure you have a copy of the handouts for each participant.

Successful Workshop

# ADVANCE PLANNER

## Presentation Information

Title _____

Date _____ Day _____ Time _____

Place _____

_____

Audience Type _____ Number _____

Purpose _____

Contact Person _____ Phone _____

## Planning Tasks

| Task | Date Completed |
|---|---|
| Contact Person(s) for Planning | |
| Confirm Date, Time & Place | |
| Make Travel & Hotel Plans | |
| Arrange for Equipment | |
| Send Workshop Agenda to Contact | |
| Personalize Workshop Outline | |
| Prepare Transparencies | |
| Prepare Handouts | |
| Other _____ | |

# ONE-HOUR WORKSHOP OUTLINE

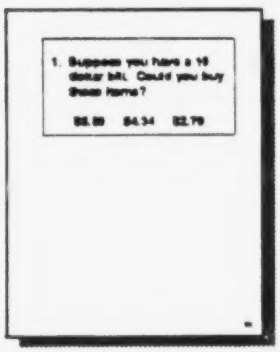
## Introduction/ Overview (15 minutes)

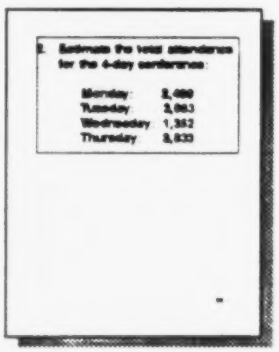
### Introduce yourself and participants

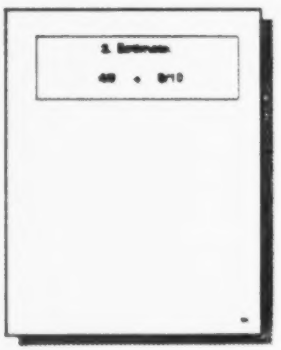
- Give a brief description of your association with Chapter 1 and describe any professional experience you have had related to Chapter 1 programs.
- If you are doing a one-hour workshop (and you have a small enough group), have paticipants briefly introduce themselves. For a three-hour workshop, askeach person to interview, then introduce another person they don't already know.
- Briefly introduce the workshop using ***Transparency T-1*** but do *not* discuss the topic of estimation in detail.

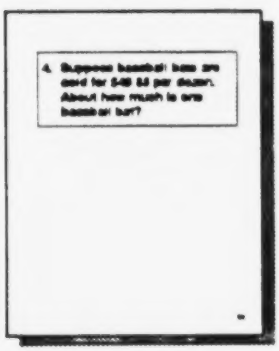
### Opening Activity ( Estimation Quiz)

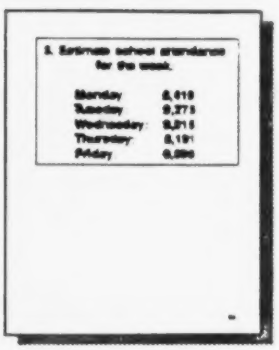
> Give the quiz using ***Transparencies (T 2-7 ).*** The quiz is designed to reveal participants' current, intuitive estimation skills and to provide background for the rest of the workshop.

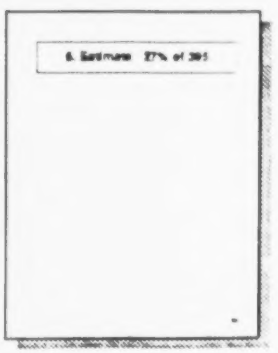












**Opening Activity** (cont.)

- Ask participants to number a blank sheet of paper from 1-6 so they will be able to record their answers. Ask them *not* to use their pencil and paper to do any computation; they should ***estimate rather than compute*** their answers.

- Display each problem on ***Transparencies 2-7*** no more than 15 seconds. Inform participants that this time constraint is to prevent the use of overly-complex strategies or

- Go over participants' answers to the quiz. Model "thinking aloud" and encourage participants to do the same. "Making your thinking public" is an important technique in mathematics instruction and should be continually modeled for participants who should, in turn, model it for their students.

**Acceptable Answer Ranges:**

1. **No (Some participants may give a range of $10-$12)**
2. **10,000 - 11,000**
3. **1.25 -1.5, or more than 1 or a little less than 1 1/2, or similar**
4. **$4.00 - $5.00**
5. **43,000 - 48,000**
6. **95 - 120**

Cover the following points while reviewing participants' answers to the quiz:

- A range of answers is always acceptable. Estimates that happen to be exact are inherently no better than other estimates within the identified range. Accepting inexact answers may be difficult for some participants, but it is necessary for developing effective estimation techniques.

- As participants give an answer, ask them to explain to the group how it was derived; then ask other members of the group for alternative methods to estimate the answer. A variety of methods for estimating should be recognized, encouraged and discussed. The practicality and ease of estimating will probably be reinforced as participants see the strategies they used in the quiz being discussed throughout the workshop.

- Reinforce the acceptability and importance of terminology participants use to describe the estimation techniques and answers; e.g, "about", "a little less than", "about half of", "almost all", "between", "close to", etc. Use the blank area at the bottom of each quiz transparency to illustrate estimation strategies and specific terminology used by participants.

**Opening Activity** (cont.)

- Be sure to remind participants that the estimation problems used in this workshop are designed to place them in the role of learner, and that these particular problems may not be appropriate for Chapter 1 students; especially those in the lower grades.

**Present the goals of the workshop**

- Display the transparency that contains the goals for the length workshop you are presenting: ***Transparency 8-A***, for the One-hour Workshop or ***8-B***, for the Three-hour Workshop.

- Discuss the goals of the workshop and how they are related to Chapter 1 mathematics instruction and the agenda for the workshop.

***Both workshops include the activities in the One-hour Workshop.*** *The three-hour workshop includes two additional hours of activities that extend the activities in the one-hour worshop.*

## One-hour Workshop

**WORKSHOP GOALS**

1. To understand estimation and its importance as part of advanced skills in Chapter 1 mathematics programs
2. To recognize classroom situations in which estimation should be used
3. To learn specific estimation strategies to use in Chapter 1 mathematics program.

The **one-hour workshop** introduces participants to research and theory on estimation in mathematics. As such, it provides the basis for the **three-hour workshop.** One hour is probably the minimum amount of time in which you can present information on estima tion in a manner that allows participants to assimilate and integrate the findings of recent research and current theory into their everyday instructional practices. Even this length workshop requires that you move efficiently through the agenda and allows only mini-mum discussion. If additional time is available, the length of the one-hour workshop can easily be expanded by using additional examples and illustrations, allowing more time for

## The "What" and "Why " of Estimation (10 minutes)

- Distribute copies of the ***Participant Handouts.*** Remind participants that most transparencies are reproduced in smaller size in the handout, so they should not feel compelled to copy what appears on the overhead projector screen.

- Present ***Transparencies 9-18*** using the notes on the following pages as a guide to the important points to cover.

---

**Estimating**

---

*Research Finding*

**Although students need to learn how to find exact answers to arithmetic problems, good math students also learn the helpful skill of estimating answers. This skill can be taught.**

*Comment*

Many people can tell almost immediately when a total seems right or wrong. They may not realize it, but they are using a math skill called estimating.

Estimating can also be valuable to children learning math.

When students can make good estimates of the answer to an arithmetic problem, it shows they understand the problem. This skill leads them to reject unreasonable answers and to know whether they are "in the ballpark."

Research has identified three key steps used by good estimators; these can be taught to all students.

- Good estimators begin by altering numbers to more manageable forms—by rounding, for example.
- They change parts of a problem into forms they can handle more easily. In a problem with several steps, they may rearrange the steps to make estimation easier.
- They also adjust two numbers at a time when making their estimates. Rounding one number higher and one number lower is an example of this technique.

Before students can become good at estimating, they need to have quick, accurate recall of basic facts. They also need a good grasp of the place value system (ones, tens, hundreds, etc.).

Estimating is a practical skill; for example, it comes in very handy when shopping. It can also help students in many areas of mathematics and science that they will study in the future.

*References*

Bestgen, B. et al. (1980). "Effectiveness of Systematic Instruction on Attitudes and Computational Estimation Skills of Preservice Elementary Teachers." *Journal for Research in Mathematics Education*. Vol. 11, pp. 124-136.

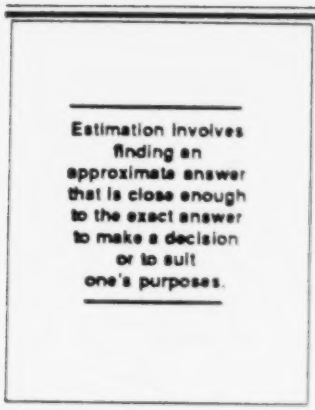
Bley, S. R. (1984). "Estimating Answers to Algebra Word Problems." *Journal of Experimental Psychology, Learning, Memory, and Cognition*. Vol. 10, pp. 778-790.

Boys, R. et al. (1982). "Processes Used by Good Computational Estimators." *Journal for Research in Mathematics Education*. Vol. 13, pp. 183-201.

Bohan, H. L. et al. (1981). "Instruction in Estimating Solutions of Whole Number Computations." *Journal for Research in Mathematics Education*. Vol. 12, pp. 165-178.

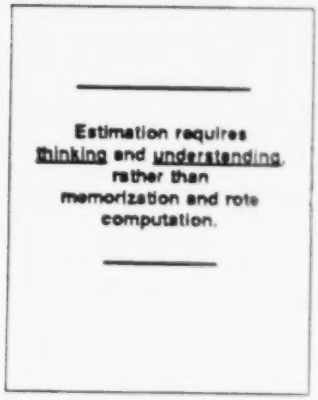
Trafton, P. R. (1978). "Estimation and Mental Arithmetic: Important Components of Computation." In *Developing Computational Skills, 1978 Yearbook*. Reston, VA: National Council of Teachers of Mathematics.

*Source:* U.S. Department of Education (1986). What Works: Research About Teaching and Learning. Washington, D.C.: U.S. Department of Education, p.31.


Estimation involves finding an approximate answer that is close enough to the exact answer to make a decision or to suit one's purposes.

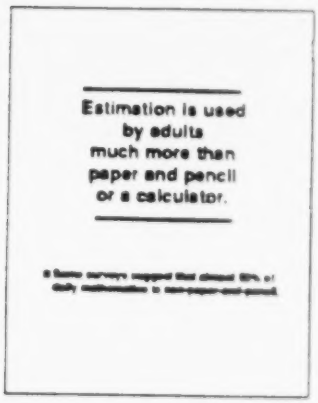
*Transparency 9*

- Emphasize "approximate answer" and "close enough "to suit one's purpose". Note that what is "close enough" will vary with each specific situation.
- Indicate that remembering a formal definition is less important than using the concepts in the definition as a guideline for effective instruction in estimation.



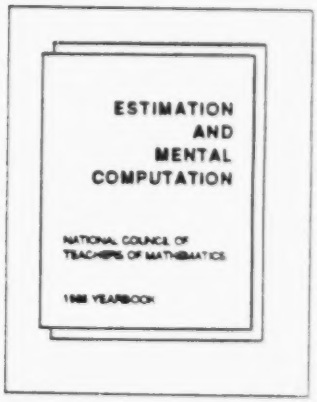

*Transparency 10*

- Note that estimation assists concept development in mathematics, an area that needs strengthening in most instructional programs.



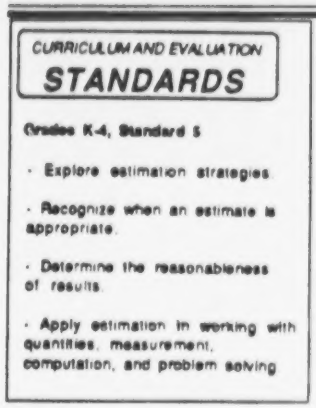

*Transparency 11*

- Mention that the reliance on pencil- and- paper computation in Chapter 1 and regular classrooms does not match the way in which most adults use mathematics.
- Estimation is more than just something to practice in school; it is a skill that is useful in the real world outside of school.



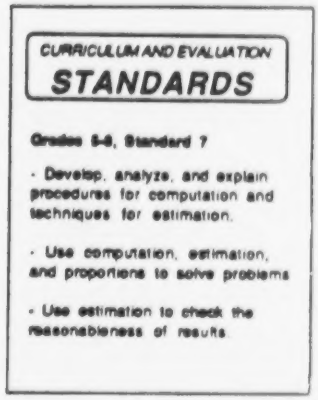

*Transparency 12*

- The legitimacy and importance of estimation in the mathematics curriculum is underscored by the National Council of Teachers of Mathematics' (NCTM) dedication of its entire 1986 Yearbook to the subject.

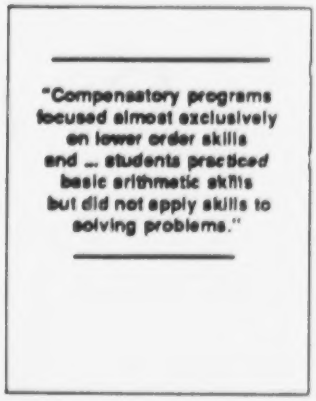

CURRICULUM AND EVALUATION
STANDARDS
Grades K-4, Standard 5
· Explore estimation strategies.
· Recognize when an estimate is appropriate.
· Determine the reasonableness of results.
· Apply estimation in working with quantities, measurement, computation, and problem solving

*Transparencies 13 and 14*

- The National Council of Teachers of Mathematics *Curriculum and Evaluation Standards* were released in March, 1989, after several years of research and development.

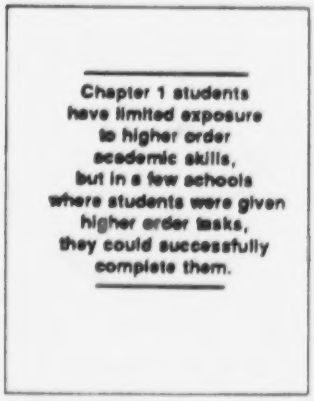

CURRICULUM AND EVALUATION
STANDARDS
Grades 5-8, Standard 7
· Develop, analyze, and explain procedures for computation and techniques for estimation.
· Use computation, estimation, and proportions to solve problems.
· Use estimation to check the reasonableness of results.

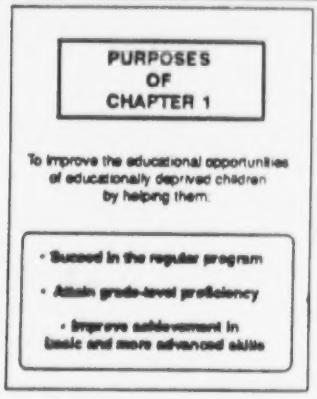
- They serve as a basis for improving the teaching and learning of mathematics in our schools. Estimation plays a key role in these new standards.

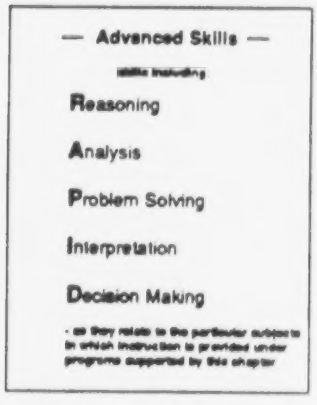

"Compensatory programs focused almost exclusively on lower order skills and ... students practiced basic arithmetic skills but did not apply skills to solving problems."

*Transparencies 15 and 16*

- In preparing for the reauthorization of Chapter 1, Congress recognized the importance of having high expectations for all students. The National Assessment of Chapter 1 conducted in 1987 indicated that many Chapter 1 programs were working exclusively on drill and practice and placing little emphasis on more challenging topics. The prevailing attitude among Chapter 1 staff members was that "the basics" must be mastered *first* . Consequently, they spent time on little else but the basics.


Chapter 1 students have limited exposure to higher order academic skills, but in a few schools where students were given higher order tasks, they could successfully complete them.


PURPOSES
OF
CHAPTER 1
To improve the educational opportunities of educationally deprived children by helping them
• Succeed in the regular program
• Attain grade-level proficiency
• Improve achievement in basic and more advanced skills




*Transparencies 17 and 18*

- In re-authorizing Chapter 1, Congress placed much emphasis on raising the expectations that *all* students can learn more advanced skills and" higher-order tasks". Improving achievement in advanced skills is now a stated purpose of Chapter 1 instructional programs.

- Advanced Skills include "R.A.P.I.D." skills: Reasoning, Analysis, Problem Solving, Interpretation, and Decision Making. Instruction in estimation will help students use these skills more effectively.

- The acronym R.A.P.I.D. has been useful for conveying the meaning of advanced skills in Chapter 1 programs.

## Participant Involvement with Estimation Strategies (20 minutes)

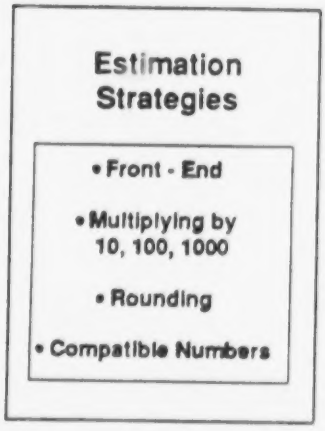
### Present computation and mental estimation as complementary skills

- Skills in *mental computation* complement estimation skills. For example, in question # 6 from the quiz participants took at the beginning of the workshop (Estimate: 27% of 395), some may have recast this problem as "about 1/4 of 400," then used mental computation to divide 400 by 4 to arrive at an estimate of 100.

- As specific problems are discussed during this part of the workshop, point out the mental computation being used. Mental computation is an excellent way to practice basic arithmetic facts and to help students understand number concepts and place value.

- Many estimation strategies can be used successfully in Chapter 1 programs. This workshop will concentrate on the four strategies presented on ***Transparency 19.*** While these strategies are presented and practiced separately in this workshop, in reality they are interdependent and are used together.

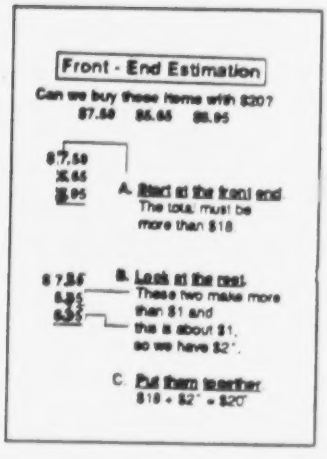
### Transparency 19




- Before the workshop, you should become familiar with the strategies on **Transparency 19.** Preview **Transparencies 20, 22, 24, 25, 27, and 28** and the **Participant Handouts.**

- The four estimation strategies are introduced separately, but they should be used flexibly. Students should be encouraged to choose among available strategies. Different students will use different strategies for the same problem. Discussing various approaches to the same problem and their advantages and disadvantages is a key to estimation instruction.

### Transparency 20

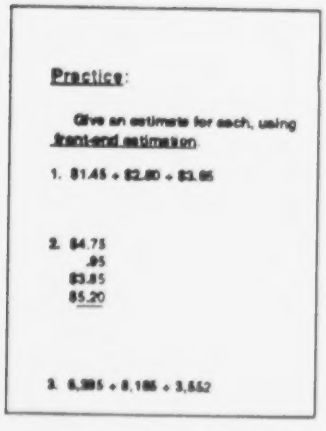

Front - End Estimation
Can we buy these items with $20?
$7.59 $5.65 $8.95
A. Start at the front end.
The total must be more than $18
B. Look at the rest.
These two make more than $1 and this is about $1, so we have $2.
C. Put them together.
$18 + $2 = $20

- Many problems that appear in mathematics textbooks can be adapted for estimating rather than finding the exact answers. Front-end estimation can also be used extensively in store and and restaurant transactions that involve money.

- The "+" and "-" are often used in estimating to denote "more than" or "less than". (For example, the "$20+" on this transparency indicates more than $20.00.) Other good estimates for this problem would be: *a little more than $20, $20.50, or $21.00.* Remember to accept a *range* of estimates and estimates closest to the exact answer are not considered better. Ask participants *not* to compute the answer; the estimate is the answer.

- Discuss how the intended use of an estimate affects how accurate it must be.

### Transparency 21

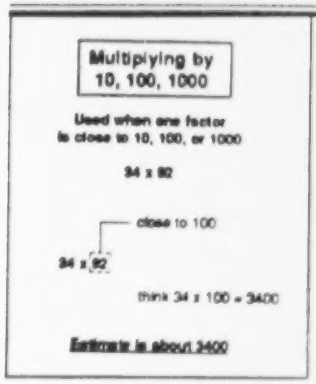



- Ask participants to write their estimates for the problems on a piece of paper instead of calling out their answers.

- They may want to copy the problem on their paper first, the write their answer; or write only their answer.

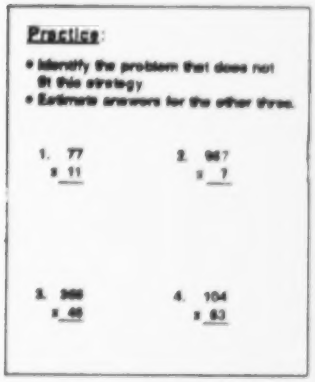
- Encourage participants to "make their thinking public" as they discuss their estimates. The *process* and the *thinking* they used is much more important than then answer.

*(Answers are not supplied for the practice problems so that a variety of estimates and strategies can be elicited from the group.)*


Multiplying by 10, 100, 1000
Used when one factor is close to 10, 100, or 1000
34 x 92
close to 100
34 x 92
think 34 x 100 = 3400
Estimate is about 3400

*Transparency 22*

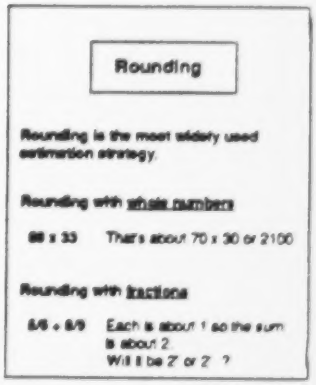
- The importance of being able to mentally compute is illustrated by the strategy of multiplying by 10, 100, and 1,000. This is a very useful strategy in a variety of situations.
- Point out that many problems do not fit this strategy. For example: *46 X 692*, because neither factor is very close to 10, 100 or 1,000. The rounding strategy (***Transparency 24***) could be used for this problem: *50 X 700*.


Practice:
• Identify the problem that does not fit this strategy
• Estimate answers for the other three.
1. 77 x 11
2. 987 x 7
3. 348 x 48
4. 104 x 83

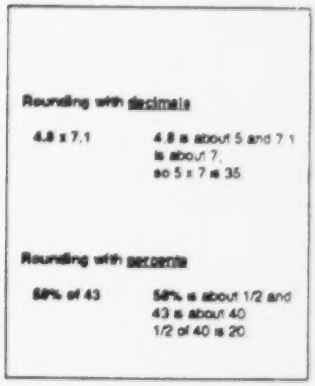
*Transparency 23*

- Use the same directions and general approach with these practice problems that you used with those on ***Transparency 21***.

*Transparency 24*

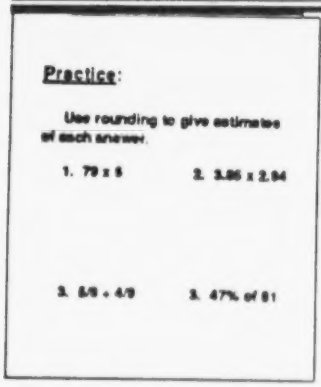

Rounding
Rounding is the most widely used estimation strategy.
Rounding with whole numbers
88 x 33 That's about 70 x 30 or 2100
Rounding with fractions
5/6 + 8/9 Each is about 1 so the sum is about 2.
Will it be 2' or 2' ?

- Rounding *for the purpose of estimation* does not have to follow the formal rules for rounding that appear in most mathematics textbooks. In estimating, the purpose of rounding is to make numbers *more manageable.*
- As you go through the rounding examples on this transparency, there is likely to be a wide range of estimates as participants use rounding techniques differently.
- In rounding fractions, it may be helpful to use such terminology as "about half" for 7/13, or "almost 1" for 8/9, or "close to 0" for 1/8. Again, emphasize that estimating is an effective way to promote understanding of a problem.


Rounding with decimals
4.8 x 7.1 4.8 is about 5 and 7.1 is about 7, so 5 x 7 is 35
Rounding with percents
58% of 43 58% is about 1/2 and 43 is about 40. 1/2 of 40 is 20

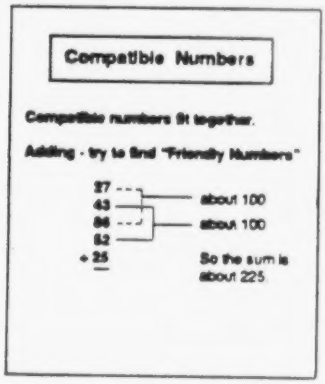
*Transparency 25*

- Estimating with fractions, decimals, and percents is an excellent way to help students improve their "number sense" in these difficult areas. Although some participants may not teach fractions, decimals, and percents in their Chapter 1 programs, exposure to estimating with these should increase their understanding. Rounding percentages is similar to rounding with fractions. Again, it is important to use terminology like "about half of" for 47%, or "almost all of" for 93%, or "a very small part of" for 6%.



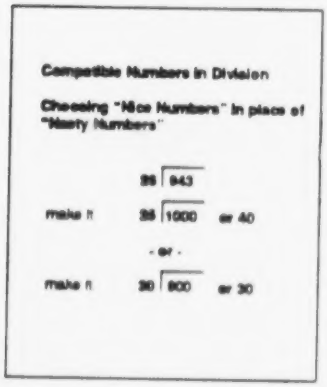

## Transparency 26

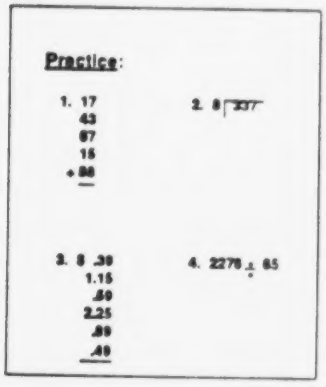
- The problems on this transparency will help participants use rounding as an estimation strategy. Although the problems may be difficult for some participants, they are intended to expand their thinking about fractions, decimals, and percents.




## Transparencies 27 and 28

- These transparencies involve compatible numbers used in addition and division.
- As with other estimation strategies, there are prerequisite skills such as being able to identify pairs of "friendly numbers" for addition and "nice numbers" for division.
- "Friendly numbers" are pairs or groups of numbers whose sum is easy to use. An example would be *62* and*43* because their sum *about 100*.
- "Nice numbers" such as *240* and*60* or *1000* and*50* can be divided without remainders.
- Using terminology like "friendly numbers", "nice numbers", and "nasty numbers" helps in teaching estimation to children.




## Transparency 29

- This transparency provides practice with the compatible numbers estimation strategy.

## Applying Estimation Strategies to Chapter 1 Instruction (15 minutes)

**Review the main points of the workshop**

« Display ***Transparency 30*** which contains a review of the main points of the workshop ».

- Remind participants that estimation should be taught as a *process* that involves thinking and understanding. It should be a naturally-occurring part of everyday classroom activities. Teachers, through effective modeling and questioning, can promote an "atmosphere for estimating" both in the regular classroom and in Chapter 1 programs.

« Show ***Transparency 31*** which contains suggestions for how participants can apply the workshop concepts and materials to their own teaching situation ».

- As you go over the points on the transparency, consider the following recommendations:

1. Start with practical situations and uses for estimation. Have students find estimates that are considered suitable answers in and of themselves, and which do not need to be followed with exact computations. Remind participants how often they, as adults, use estimation in their everyday lives. Encourage them to share their own experiences in using estimation with their students.

2. Use the language of estimating. Estimating activities provide an excellent way for Chapter 1 instructors to help promote students' understanding and number sense. Asking students questions like "About how many ...?" or "Will it be larger than or smaller than ...?" will create an atmosphere for estimation in the classroom.

3. Make time for estimating in the Chapter 1 classroom. An exercise frequently found in mathematics textbooks asks students first to estimate, then find the exact answer. When learning estimation strategies, students often have difficulty understanding the purpose of estimation since they know they will have to compute the exact answer anyway. Estimation activities should be substituted for some of the usual pencil-and-paper computational exercises. Almost any lesson, exercise or worksheet can easily be adapted to include estimation. Estimation is best used to promote discussion and understanding. Guard against making estimation an added assignment or burden to students.

« Display***Transparency 32*** which contains the final activity in which participants will select a typical lesson from pp. 9-15 of the ***Participant Handouts*** and adapt it for use in their own teaching situation. »

- Participants should choose one of the activities on **H 9-15** of the handout and devise a specific plan for how to incorporate estimation strategies into the activity. The five questions on ***Transparency 32*** and on the top of **H 9-15** of the handout should help participants adapt the activity they selected to include estimation strategies. *(H-9a displays a "model" plan.)*

- Every effort should be made to get all participants actively involved in this final activity in which they will *apply* what they have learned in this workshop to their own teaching situation. This activity is designed to help participants bridge the gap between what they learned in the workshop and what they will be doing in their Chapter 1 programs.

- If time permits, participants should work in groups that have chosen the same activity to adapt and share their ideas about incorporating estimation strategies into the activity. This type of small-group activity is used in the three-hour workshop format.

««Show ***Transparency 33*** which contains a 3-week log for recording estimating activities in the classroom.»»

- Logging estimation activities serves as a motivational device for the instructor and students to continue daily estimation activities and provides a written record to refer to at a later time. ***(H-16 in the handout.)***

## Summary and Conclusion

**Encourage participants to try to use at least one "personalized" estimation activity per day with each student in the Chapter 1 program.** This is one of the best ways to promote understanding in mathematics and provide students with a useful, life-long skill.

**If you are doing *only* a one-hour workshop, conclude the workshop now.**

1. Provide information on other workshops you are willing to do and tell participants how to contact you for further information or to schedule workshops.

2. Thank participants for their attention and involvement and invite any who have questions or things they would like to discuss to stay.

**If you are doing a three-hour workshop, this would be an excellent time for a break.**

# Section 2:

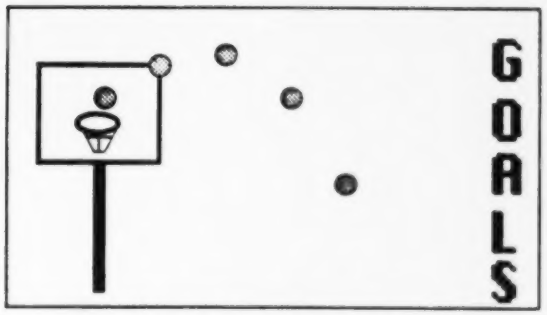
# PARTICIPANT HANDOUTS

resource CIC center

Chapter 1
Curriculum and Instruction
Resource Center

# Advanced Skills for Chapter 1 Mathematics: ESTIMATION

## Participant Handouts




## Participants will:

- Understand estimation and its importance as part of advanced skills in Chapter 1 mathematics instruction.
- Recognize classroom situations in which estimation should be used.
- Learn specific estimation strategies and activities to use in Chapter 1 mathematics programs.

T-8

---

**Estimation involves finding an approximate answer that is close enough to the exact answer to make a decision or to suit one's purposes.**

---

T-9

---

**Estimation requires thinking and understanding, rather than memorization and rote computation.**

---

T-10

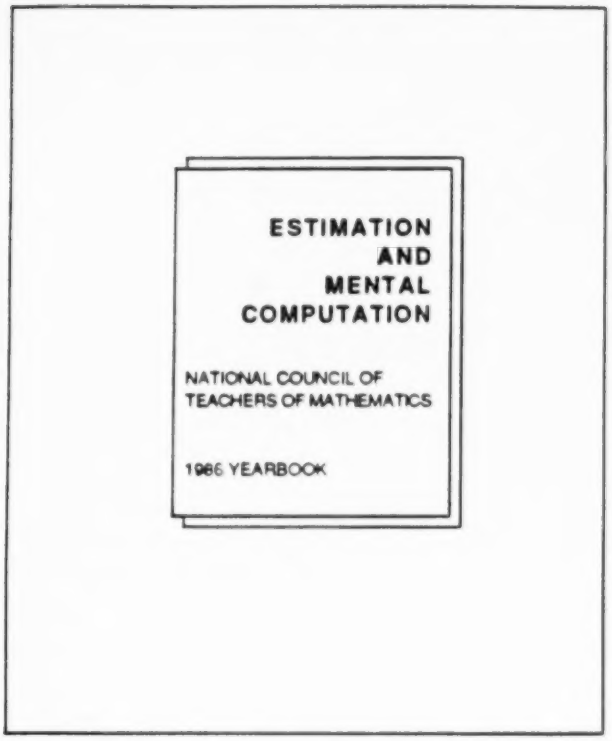
---

**Estimation is used by adults much more than paper and pencil or a calculator.**

---

- Some surveys suggest that almost 90% of daily mathematics is non-paper-and-pencil.

T-11




T-12

*CURRICULUM AND EVALUATION*

## *STANDARDS*

**Grades K-4, Standard 5**

- Explore estimation strategies.
- Recognize when an estimate is appropriate.
- Determine the reasonableness of results.
- Apply estimation in working with quantities, measurement, computation, and problem solving.

Source
Curriculum and Evaluation Standards NCTM, 1989

T-13

*CURRICULUM AND EVALUATION*

## *STANDARDS*

**Grades 5-8, Standard 7**

- Develop, analyze, and explain procedures for computation and techniques for estimation.
- Use computation, estimation, and proportions to solve problems.
- Use estimation to check the reasonableness of results.

Source
Curriculum and Evaluation Standards NCTM, 1989

T-14

**"Compensatory programs focused almost exclusively on lower order skills and ... students practiced basic arithmetic skills but did not apply skills to solving problems."**

Source
The Current Operation of the Chapter 1 Program
National Assessment of Chapter 1, 1987

T-15

**Chapter 1 students have limited exposure to higher order academic skills, but, in a few schools where students were given higher order tasks, they could successfully complete them.**

Source
The Whole School Day Study, Rowan, Guthrie, Lee and Guthrie, 1986

T-16

## PURPOSES OF CHAPTER 1

To improve the educational opportunities of educationally deprived children by helping them:

- **Succeed in the regular program**
- **Attain grade-level proficiency**
- **Improve achievement in basic and more advanced skills**

T-17

## — Advanced Skills —

**skills including:**

**R**easoning

**A**nalysis

**P**roblem Solving

**I**nterpretation

**D**ecision Making

**- as they relate to the particular subjects in which instruction is provided under programs supported by this chapter.**

Source
Section 1471 of the Augustus F. Hawkins-Robert T. Stafford Elementary and Secondary School Improvement Amendments of 1988

T-18

## Estimation Strategies

- **Front - End**
- **Multiplying by 10, 100, 1000**
- **Rounding**
- **Compatible Numbers**

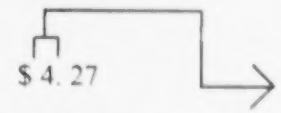
T-19

## Front-End Estimation

Front-end estimation is useful when doing "store math" or "restaurant math."

**$ 4. 27 + 1. 92 + 2.07**

**A. Start at the<u>front end</u>.**

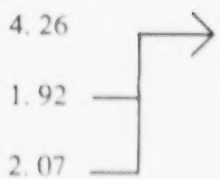


$ 4. 27 **The total must be more than $7.**

1. 92

\+ 2. 07

**B. Look at the rest.**



4. 26 **92 ¢ + 7 ¢ is close to $1.**

1. 92 **26 ¢ makes it go over $1.**

2. 07

**C. Put them together.**

**Estimate $ 7 and $1 = $8+**

The range of estimates would probably be between $ 8 and $ 8.50.
(with tax $ 9 is an acceptable estimate)

**Practice**

You have $10.00. Can you buy each group of items?

1. $ 7. 19 and $1. 98

2. $ 4. 26
   2. 96
   3.52

3. $ 3. 07 $ 1. 32 $ 1. 78 $ 3. 19

## Multiplying by 10, 100, 1000

Used when one factor is close to 10, 100, 1000

**96 x 63 =**

96 is close to 100, so our estimate is 100 x 63 or 6300.

To use this estimation strategy, children must be able to multiply by 10, 100, or 1000 using mental computation. Be sure students understand and receive practice with this skill.

**Sample Practice Exercises**

| | x10 | x100 | x1000 |
|---|---|---|---|
| 6 | 60 | _____ | _____ |
| 38 | _____ | 3800 | _____ |
| 243 | _____ | _____ | 243,000 |
| 609 | _____ | _____ | _____ |

**Extending the strategy**

We can make a closer estimate by adjusting our original estimate of 6300. Since 96 was rounded up to 100, our answer is an overestimate. Adjusting it lower --to 6200, for example--will allow us to arrive at a better estimate.

**Practice**

Estimated answers for four of these problems can be easily found by using the "Multiply-by-10,100,1000" strategy. Two of the problems do not work well with the strategy. Find estimates for four of the problems.

1. 692 x 6

2. 93 x 4

3. 62 x 98

4. 1,017 x 7

5. 724 x 104

6. 56 x 72

## Rounding in Estimation

Rounding is the most widely used estimation strategy. It has application to whole numbers, fractions, decimals, and percents.

- **Multiplying whole numbers**

  63 x 46 — That's about 60 x 40 or 2400.
  or
  That's about 60 x 50 or 3000.

- **Multiplying decimals**

  6.1 x 2.8 — 6.1 is about 6 and 2.8 is about 3. So 6 x 3 is 18.

- **Cleaning up fractions**

  $\frac{7}{8} + \frac{11}{12}$ — Each is about 1, so the sum is about 2. Will it be 2+ or 2-?

- **Percents**

  46% of 83 — 46% can be thought of as about 1/2, and 83 can be rounded to 8.0. So 1/2 of 80 is about 40.

**Practice**

Use rounding to give estimates.

1. 93% of 60

2. $\frac{1}{9} + \frac{4}{7} + \frac{3}{8}$

3. 47.8 x 3.2

4. 84 x 68

## Compatible Numbers
## (Used for addition and division)

Compatible numbers are numbers that fit together and make estimating easier. The term "compatible" is not suggested for use with students.

For addition, use the term "Friendly Numbers" for pairs or groups of numbers whose sum is easy to work with.

**Example:**

$$\begin{array}{r} 24 \\ 43 \\ +78 \\ \hline \end{array} \rightarrow$$

24 and 78 are "friendly" because there sum is about 100. An estimate for the problem is about 143 or 150.

**Example:** (upper grade)

$$\begin{array}{r} 2\frac{}{}5 \\ 1\frac{}{}6 \\ +7\frac{}{}8 \\ \hline \end{array} \rightarrow$$

These two fractions are "friendly" because their sum is about 1. An estimate for the problem is about 1 2/5 or 1 1/2.

For division, use the term "Nice Numbers." "Nice Numbers" are substituted for "Nasty Numbers" when making division estimates.

**Example:** $36\overline{)295}$ Substitute "nice numbers" $40\overline{)280}$ or about 70.

Other "nice numbers" $30\overline{)270}$ or about 90.

Note that the two division estimates vary considerably. These would be in an acceptable range for most uses. Division is a difficult concept. Estimating activities using "nice numbers" will add to student's understanding of the division process.

**Practice**

Use "friendly" or "nice numbers" to estimate each of the following:

1. $\begin{array}{r} 48 \\ 93 \\ 77 \\ 13 \\ +55 \\ \hline \end{array}$

2. $5\overline{)211}$

3. $\begin{array}{r} 6\frac{}{}11 \\ 1\frac{}{}9 \\ 4 \\ +10 \\ \hline \end{array}$

4. $6284 \div 82$

H-7

## Teaching Estimation as a Process

- Include estimating and estimates as part of every math lesson.
- Teach and model estimation strategies.
- Accept and model different strategies for the same problem.
- Accept and discuss a range of reasonable estimates.
- Teach mental computation along with estimation.

T-30

## Getting Started

- Start with practical applications.
- Don't start by having students estimate, then compute exact answers.
- Model the use of estimation terminology – ask a lot of estimation questions.
- Make time by substituting estimation activities for paper-and-pencil computation.
- Involve each Chapter 1 student with an estimation activity each day.

T-31

## Adapting a Lesson

1. Will this be an introductory estimation lesson or an application of a strategy already taught?
2. Will the entire lesson be focused on estimation or will estimation be only part of the lesson?
3. Will a specific strategy be emphasized or will students choose?
4. Are there mental computation skills that need to be taught or reviewed to effectively use the strategy?
5. Which problem(s) will be used and how?

T-32

## *3-WEEK "ESTIMATING" LOG*

What did I do with my students each day to promote estimation? Write down your activity. It will serve as a motivator to continue and a resource for later.

**WEEK 1**

Monday

Tuesday

Wednesday

Thursday

Friday

**WEEK 2**

Monday

Tuesday

Wednesday

Thursday

Friday

**WEEK 3**

Monday

Tuesday

Wednesday

Thursday

Friday

H-16

## ADAPTING A LESSON

Adapt the following lesson to include estimation language and strategies. The following questions may be helpful in guiding your planning.

1. Will this be an introductory estimation lesson or an application of a strategy(s) already taught?

2. Will the entire lesson be focused on estimation or will estimation be only part of the lesson?

3. Will a specific strategy(s) be emphasized or will students choose their own?

4. What mental computation skills need to be taught or reviewed to effectively use the strategy(s)?

5. Which problem(s) will be used? How will they be used?

***EXERCISES*** ***Add. Check answers.***

| | | | |
|---|---|---|---|
| 1. 4<br>5<br>+8 | 2. 7<br>3<br>6<br>+5 | 3. 64<br>98<br>+52 | 4. 48<br>27<br>65<br>+17 |
| 5. 471<br>526<br>+212 | 6. 148<br>364<br>+534 | 7. 7,584<br>1,657<br>+4,798 | 8. 6,458<br>984<br>+1,326 |
| 9. 496<br>745<br>377<br>+634 | 10. 8,457<br>890<br>6,729<br>+ 384 | 11. $24.85<br>63.32<br>39.72<br>+ 54.28 | 12. $48.64<br>76.95<br>35.79<br>+ 2.85 |

13. 73 + 439 + 48 + 236

14. 9,684 + 2,387 + 575

*Solve.*

15. Jim earned $39.52, $54.87, and $49.67. How much did he earn?

16. Ann read three books with 264, 98, and 174 pages. Find the total number of pages she read.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# ADAPTING A LESSON

Adapt the following lesson to include estimation language and strategies. The following questions may be helpful in guiding your planning.

1. Will this be an introductory estimation lesson or an application of a strategy(s) already taught? Flexibly using strategies already introduced.

2. Will the entire lesson be focused on estimation or will estimation be only part of the lesson? Combination of mental computation, estimation, and computation.

3. Will a specific strategy(s) be emphasized or will students choose their own?
   - front end
   - rounding
   - friendly numbers

   } Discuss and reinforce the flexible use of these 3.

4. What mental computation skills need to be taught or reviewed to effectively use the strategy(s)? Nothing special.

5. Which problem(s) will be used? How will they be used?
first Ex. 1-3 mental computation – exact answers
second Ex. 4-6 students estimate, then class discusses
third Ex. 8 Is this a good estimate? Why or why not?
fourth Ex. 9-16 Assignment – Students choose 4 to estimate and 4 to compute.

***EXERCISES*** ***Add. Check answers.***

first — mental comp.

1. 4 / 5 / +8
2. 7 / 3 / 6 / +5
3. 64 / 98 / +52
4. 48 / 27 / 65 / +17

second — Students estimate using whatever methods they choose. Discuss.

5. 471 / 526 / +212
6. 148 / 364 / +534
7. 7,584 / 1,657 / +4,798
8. 6,458 / 984 / +1,326 / 7,000

third — Is this a good estimate?

9. 496 / 745 / 377 / +634
10. 8,457 / 890 / 6,729 / + 384
11. $24.85 / 63.32 / 39.72 / + 54.28
12. $48.64 / 76.95 / 35.79 / + 2.85

fourth — assignment students choose 4 to estimate and 4 to compute exact answers.

13. 73 + 439 + 48 + 236
14. 9,684 + 2,387 + 575

***Solve.***

15. Jim earned $39.52, $54.87, and $49.67. How much did he earn?
16. Ann read three books with 264, 98, and 174 pages. Find the total number of pages she read


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H-9a

## ADAPTING A LESSON

Adapt the following lesson to include estimation language and strategies. The following questions may be helpful in guiding your planning.

1. Will this be an introductory estimation lesson or an application of a strategy(s) already taught?

2. Will the entire lesson be focused on estimation or will estimation be only part of the lesson?

3. Will a specific strategy(s) be emphasized or will students choose their own?

4. What mental computation skills need to be taught or reviewed to effectively use the strategy(s)?

5. Which problem(s) will be used? How will they be used?

***EXERCISES*** ***Subtract. Check your answer.***

| | | | | | |
|---|---|---|---|---|---|
| 1. 75 − 34 | 2. 87 − 53 | 3. 63 − 23 | 4. 44 − 19 | 5. 94 − 68 | 6. 73 − 25 |
| 7. 657 − 234 | 8. 786 − 239 | 9. 464 − 285 | 10. 562 − 478 | 11. 945 − 678 | |

12. 38 − 24 13. 71 − 45 14. 364 − 129 15. 836 − 378

***Complete.***

16. $4\square - \square 7 = 12$

17. $\square 6 - 38 = 2\square$

18. $\square 9 - 6\square = 25$

19. $6\square - 25 = \square 6$

***Solve.***

20. There are 32 boys and 25 girls in the fifth grade. How many more fifth-grade boys are there than fifth-grade girls?

21. Acme Corporation has 326 employees. Of these, 172 employees are on vacation. How many employees are still working?

23

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## ADAPTING A LESSON

Adapt the following lesson to include estimation language and strategies. The following questions may be helpful in guiding your planning.

1. Will this be an introductory estimation lesson or an application of a strategy(s) already taught?

2. Will the entire lesson be focused on estimation or will estimation be only part of the lesson?

3. Will a specific strategy(s) be emphasized or will students choose their own?

4. What mental computation skills need to be taught or reviewed to effectively use the strategy(s)?

5. Which problem(s) will be used? How will they be used?

***Solve. Use the chart.***

| Library Supply Store | |
|---|---|
| Books About Jobs | $ 5.99 |
| Picture Books | 10.95 |
| Sports Books | 16.50 |
| Books About Pets | 3.65 |
| Mystery Books | 8.75 |
| Hobbies and Crafts | 12.95 |

1. What is the cost of 3 books about sports?

2. How much would 5 books about hobbies and crafts cost?

3. What is the cost of 4 picture books?

4. How much would it cost to buy 10 books about pets?

5. Mrs. O'Brien wants to buy 3 mystery books and 4 books about jobs. How much will she have to pay?

6. What is the difference in price between 2 books on sports and 3 picture books?

7. Mrs. James had $50 to spend for books. She bought 3 books on hobbies and crafts. How much does she have left?

8. The store has 6 different books about pets: Dogs, Cats, Birds, Fish, Rabbits, and Hamsters. Billy wants to buy books about two different pets. How many different pairs of pet books could he select?



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H-11

## ADAPTING A LESSON

Adapt the following lesson to include estimation language and strategies. The following questions may be helpful in guiding your planning.

1. Will this be an introductory estimation lesson or an application of a strategy(s) already taught?

2. Will the entire lesson be focused on estimation or will estimation be only part of the lesson?

3. Will a specific strategy(s) be emphasized or will students choose their own?

4. What mental computation skills need to be taught or reviewed to effectively use the strategy(s)?

5. Which problem(s) will be used? How will they be used?

***EXERCISES*** ***Solve. Use the chart.***

1. The art teacher ordered 23 packages of construction paper. Find the total cost.

**School Supplies Store**

| | |
|---|---|
| Box of 100 pencils | $3.59 |
| Package of 500 sheets of writing paper | $4.07 |
| Box of 20 crayons | $1.35 |
| Package of 50 sheets of construction paper | $6.50 |

2. Mr. Sesher ordered 30 packages of writing paper. How many sheets will he get?

3. The student council ordered 40 boxes of pencils to sell. How many pencils were ordered?

4. The kindergarten teacher needed to buy 18 boxes of crayons. Find the total cost.

5. Find the cost of 28 packages of writing paper.

6. How many crayons would the school have if 40 boxes were ordered?

7. The school ordered a package of construction paper for each of the 200 students. How many sheets would that be?

8. Find the cost of 16 boxes of pencils.

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# ADAPTING A LESSON

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2. Will the entire lesson be focused on estimation or will estimation be only part of the lesson?

3. Will a specific strategy(s) be emphasized or will students choose their own?

4. What mental computation skills need to be taught or reviewed to effectively use the strategy(s)?

5. Which problem(s) will be used? How will they be used?

***EXERCISES*** ***Multiply. Check your answers.***

1. $52 \times 38$

2. $48 \times 23$

3. $264 \times 37$

4. $39 \times 41$

5. $62 \times 38$

6. $51 \times 72$

7. $72 \times 63$

8. $58 \times 64$

9. $79 \times 67$

10. $604 \times 85$

11. $\$7.28 \times 32$

12. $61 \times 35$

13. $45 \times 38$

14. $44 \times 325$

15. $65 \times \$3.62$

**Solve. Use the chart.**

16. How many apples were there the first year? the fifth year?

| Gain in the Amount of Apples | | |
|---|---|---|
| | First Year | Fifth Year |
| trees | 54 | 81 |
| average number of apples on each tree | 658 | 863 |

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## ADAPTING A LESSON

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3. Will a specific strategy(s) be emphasized or will students choose their own?

4. What mental computation skills need to be taught or reviewed to effectively use the strategy(s)?

5. Which problem(s) will be used? How will they be used?

***EXERCISES*** ***Tell if the first digit you write in the quotient is in the hundreds, tens, or ones place.***

1. $64\overline{)819}$ 2. $39\overline{)237}$ 3. $93\overline{)9,764}$ 4. $43\overline{)3,864}$

***Divide. Check your answers.***

5. $79\overline{)474}$ 6. $36\overline{)284}$ 7. $61\overline{)737}$ 8. $53\overline{)386}$

9. $45\overline{)3,375}$ 10. $24\overline{)1,647}$ 11. $87\overline{)17,661}$ 12. $34\overline{)26,728}$

13. $23\overline{)7,564}$ 14. $68\overline{)4,862}$ 15. $25\overline{)18,647}$ 16. $89\overline{)75,461}$

***Estimate.***

17. $56\overline{)6,482}$ ➧ $60\overline{)6,000}$ 18. $42\overline{)3,592}$ ➧ $40\overline{)3,600}$

19. $31\overline{)927}$ 20. $67\overline{)1,517}$ 21. $15\overline{)3,943}$ 22. $54\overline{)19,480}$

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H-14

## ADAPTING A LESSON

Adapt the following lesson to include estimation language and strategies. The following questions may be helpful in guiding your planning.

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3. Will a specific strategy(s) be emphasized or will students choose their own?

4. What mental computation skills need to be taught or reviewed to effectively use the strategy(s)?

5. Which problem(s) will be used? How will they be used?

| | | |
|---|---|---|
| Round Steak | 1 lb | $1.98 |
| Rib Eye Steak | 1 lb | $2.99 |
| Whole Chicken | 1 lb | $0.59 |
| Corn | 10 ears | $1.19 |
| Peaches | 5 lb | $2.99 |
| Watermelon | 2 for | $4.89 |

***Solve. Use the ad.***

1. What is the cost of 1 watermelon?

2. What is the cost of 1 ear of corn?

3. What is the cost of 3.5 pounds of round steak?

4. What is the cost of 4 pounds of rib eye steak?

5. Mr. Babb bought 10 pounds of chicken for the freezer. What was the cost?

6. Mr. Babb wants 50 ears of corn for the freezer. How much will the corn cost?

7. Mrs. Ford needs 3 watermelons for a party. What will they cost?

8. Mrs. Harris bought 3 pounds of peaches. How much did they cost?

9. Mrs. Brown bought 5 ears of corn, 2.8 pounds of round steak, and a watermelon. How much was her bill?

10. Mrs. Ford wants to know which will cost more, 3.5 pounds of round steak or 2.7 pounds of rib eye steak. Help her decide.

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# 3-WEEK "ESTIMATING" LOG

**What did I do with my students each day to promote estimation? Write down your activity. It will serve as a motivator to continue and a resource for later.**

**WEEK 1**

Monday

Tuesday

Wednesday

Thursday

Friday

**WEEK 2**

Monday

Tuesday

Wednesday

Thursday

Friday

**WEEK 3**

Monday

Tuesday

Wednesday

Thursday

Friday

H-16

# BACKGROUND PAPER

**Promoting Educational Change:**

As professional educators we realize that the school systems in which we operate reflect our local and national priorities. We also know that education has historically been a result and outcome of the Industrial Age in which "every educational system...wants to produce a certain kind of human being" (Bloom, The Closing of the American Mind). To produce the "certain kind" of individual, education has operated with two primary purposes:

- to culturally imprint the society's youth and
- to promote and provide opportunities for students' individual self-fulfillment.

Both of these purposes are still applicable in our current age which is changing from an industrial to a technological and informational base. The world has become a global society based on rapidly changing communication and computer technology. The" Information Age" requires that individuals be mathematically literate and have acquired lifelong learning, that opportunity be provided for all, and that its electorate be informed (NCTM, 1989).

With the increasing demands placed upon education the question arises -- How can we possibly bridge the gap between old and new philosophies and still prepare our students for their currently changing life skills and the future workplace? We can continue to recognize that individuals who have developed self-esteem through successful experiences are more productive and self-fulfilled. We can recognize that to be retained, information must be relevant to the learner's life-experience. As advocates for our students, we have also long asserted that each individual learner has the right to expect personal and individual attainment of excellence in education. And, as asserted by the National Commission of Excellence in Education, such attainment requires performing on the boundary of individual ability by testing and pushing back personal limits, both in school and in the workplace.

The Commission's recommendations that schools set high expectations and goals for all learners strengthens the position of "resource professionals" such as Chapter 1 staff. But a study by the National Assessment of Chapter 1 (1987) included the following:

While . . .

- Almost 90% of everyday mathematics is performed without pencil and paper, and

- Success of workplace and everyday mathematics depends upon sound concepts of number and the ability to utilize thinking and problem-solving skills.

Yet . . .

- Some researchers in the Chapter I study felt that "current approaches to compensatory education systematically under-challenge disadvantaged students and that continual drill and repetition of basic skills limits Chapter I students opportunities to progress to more challenging material" (p.86).

- Researchers also found that although Chapter 1 students may have limited exposure to higher-order academic skills such as problem solving, students given higher-order tasks could successfully complete them (Rowan, et. al., 1986).

- The Chapter I Law of 1988 (Section 1001 (b)) requires that in addition to attaining grade-level proficiency and regular classroom achievement, program students must learn basic and "more advanced" skills.

How can educators maintain current curriculum requirements, address the Commission recommendations, the research findings, the Chapter I Law, and still adequately prepare students? The most obvious strategy is to study much of the same mathematics but, with a different emphasis, and to include of problem-solving skills. The introduction of estimation and mental computation into the teaching methodology presents exciting possibilities for both educators and students.

By such inclusion, we are meeting the Chapter I mandate to also teach the "more advanced" skills which include "reasoning, analysis, problem solving, interpretation, and decisionmaking (RAPID)". We are introducing skills that will serve our students as lifelong abilities. We are equipping them for success, not only in the regular classroom but in everyday situations. We are enabling them to utilize their education as an ongoing resource. And we are promoting the abilities to make decisions, reason, analyze, hypothesize, interpret, and think independently.

By teaching estimation and mental compuation, we are also creating an environment with emphasis on individual choice (and, thus, individual worth), strategy development and discussion. We are creating an environment with potential for students to learn, to apply more advanced skills, and to become mathematically and educationally literate.

**Estimating is an everyday skill.**

Although seldom identified as estimation skills, a variety of strategies are employed by adults everyday, for example, in answering such questions as:

- How much will these line items cost?

- How soon can we meet the deadline?

- How long will it take to get there?
- Will there be enough?
- How many should we buy?
- Will everything fit into the trunk?

As noted by Suydam (1984) "Most people use estimation every day -- more often than they do paper-and-pencil computation. Thus, computational estimation textbook analyses indicate that it has been one of the most neglected topics in the curriculum. It is, therefore, no surprise that students perform poorly on estimation questions in national assessments."

"Common sense" approaches to estimating and mental calculating require skills that are sometimes overlooked in classroom situations, but are often the most useful mathematical skills. Estimating allows an individual to decide if a calculation is reasonable, and mental computation can save both time and effort. Mastering these skills requires more, however, than learning a few new "tricks". Acquiring a real sense of number, an understanding of operations, a feeling for spatial relationships and measurement is fundamental in effectively applying estimation skills.

**How is estimation defined?**

Estimation involves using mental skills to find an approximate answer in order to make a decision or to suit a particular purpose. (Estimation also includes two distinct subcategories: measurement estimation and computational estimation.) Mental computation usually results in one answer while there are *many* correct answers to an estimation exercise. With both skills, a "variety of methods or techniques can be used on each exercise" which are also orally discussed as a part of the classroom activity. In turn, the discussion "provides an excellent opportunity for students to develop communication skills in the mathematics area"(Grouws, 1989).

**The importance of teaching estimation is well documented.**

The *Curriculum and Evaluation Standards for School Mathematics* recently published by the National Council of Teachers of Mathematics (NCTM) includes estimation as one of its curriculum standards for grades K-4 and 5-8. Implicit in the placement of estimation in the early, formative years is the fundamental importance of estimation skills in later grades and in real-life situations. Although estimation is not explicitly stated as a standard for grades 9-12, estimation and its related concepts are found in many of the other standards.

The *Standards'* emphasis on estimation skills is reinforced by researchers who have identified three "key" processes used by competent estimators:

1) Reformulation: the process of changing information to a form more easily managed mentally (e.g., rounding, using front-end or left-most numbers, substitution). Such numbers are relatively close to the original but are easier to work with mentally.

2) Translation: the process of changing the structure of a mathematical problem to a form more easily managed mentally.

3) Compensation: making adjustments which reflect variations that resulted from using reformulation or translation. So the actual (exact) answer may be less or greater than the estimation (Reys, et. al., 1982).

The Reys study also found that competent estimators have the following characteristics:

- quick and accurate recall of basic facts,
- understanding of place values,
- ability to change data to a mentally managed form (reformulation),
- ability to do mental computations quickly,
- tolerence for error,
- ability to translate a problem to a more mentally manageable form (translation),
- ability to adjust an initial estimate (compensation), and
- knowledge of properties of numbers and order of operations.

**Learning estimation strategies and their flexible use is important.**

The advantages of teaching estimation were documented in a 1981 study and include the following:

- There was no adverse effect when computational drill was replaced by estimation instruction.

- Estimation strategies can actually be taught in a short time period.

- Pupils not only became more adept estimators but adopted a valid strategy.

- Competent estimators could usually judge their estimation in comparision to the exact answer. Students could also note whether they had rounded to higher or lower number (Schoen, et.al., 1981).

While the study utilized fourth, fifth, and sixth grade students, these advantages can certainly be applicable to all grade levels.

The advantage for both teacher and student is that estimation and mental computation can be taught through a problem-solving approach. Because such skills can be processed through a variety of individual methods, requiring complete mastery of particular strategies (i.e., rounding) is not a pre-requisite, nor do such strategies have to be taught separately (Grouws, 1989). The approach to teaching estimation is not to introduce a single strategy or to practice a single strategy by working with large numbers of problems. And, rather than leading to rigidity in learning, the teaching of estimation focuses on introducing several strategies and using these flexibly with a variety of topics.

**Estimation strategies and their definitions:**

As identified by researchers, estimation skills center around four broad strategies. In each of these, be aware of the important role of language and the use of such terms as "at least, at most, either-or, both-and, if-then; more, more than, about, close to, closer to, near, between; under, over, bigger, smaller, fewer," and "a little less than."

FRONT-END:

Begin with the digit representing the largest place value, that is, the "front-end" or left-most digit.

Mentally compute the sum of the "front-end" digits, using appropriate place values.

Adjust by considering the value of the remaining digits.

Remember that this process will always produce an underestimate. However, the adjustment step provides the greater accuracy.

*Example:*

| | |
|---|---|
| 4 X 736 ... | 4 X 700 is about 2800 (front-end).<br>4 X 36 is about 100 (adjustment).<br>So, 4 X 736 is about 2900. |

This strategy works well with whole numbers, fractions, and decimals. Modified, the strategy applies in all the of the four math operations, although its strongest application is in addition (Reys, B., 1986).

ROUNDING:

Used most often for estimating the product of two multidigit factors, rounding involves three choices: rounding up, rounding down, or rounding both factors in opposite directions.

In the rounding process, numbers are first rounded. Secondly, the rounded numbers are computed. An adjustment step can be added when both factors are rounded in the same direction but is unnecessary when factors are rounded in the opposite directions.

*Example*:

| | |
|---|---|
| 23 X 78 . . . | about 20 X 80 or 1600 |
| | more than 20 X 78 or 1560+ |
| | more than 20 X 70 or 1400+ |
| | about 25 X 80 or 2000. |

This strategy strengthens "the understanding that estimation is a simplifying process" and that students must learn flexibility in order to make their method fit particular situations, operations, and numbers involved (Reys, B., 1986).

MULTIPLYING BY 10, 100, 1000:

Used in mental computation when a factor is close to 10, 100, or 1000, this strategy requires that students understand the basic concept of multiplying by these numbers and receive practice with this skill as mental computation.

*Example:*

| | |
|---|---|
| 43 X 2 | 43 is close to 40 |
| | So 43 X 2 is about 80. |
| 46 X 2 | 46 is closer to 50 |
| | So 46 X 2 is about 100. |

COMPATIBLE NUMBERS:

The strategy refers to a set of numbers that can be easily fit together and encourages the student to consider the relationship of the entire problem by looking at all the numbers involved in the problem and changing or rounding each in order to pair one number with another.

*Example*:

| | |
|---|---|
| Add | 27 |
| | 38 |
| | 65 |
| | 81 |
| Look for hundreds... | |
| 38 | 27 |
| 65 | 81 |
| The sum is about 200. | |

Most effective when estimating division problems, the compatible number strategy can also be used for addition problems with several addends (Reys, B., 1986).

**Bridging the gap between the workshop and the classroom.**

The purpose of the Estimation Workshop is to promote the teaching and learning of advanced skills in Chapter 1 mathematics programs. Traditionally, both Chapter 1 and regular classroom instruction has centered around the learning of basic facts and computational skills. The National Council of Teachers of Mathematics (NCTM) has urged that instruction focus on problem solving. The National Assessment of Educational Progress (1987) notes that "simple drill does not alleviate student weakness in understanding the subject matter." What is needed is mathematics instruction that prepares students for a rapidly changing technological world. Students need improved problem-solving abilities and increased capability to use many new skills such as mental computation, data gathering, and estimation (Grouws, 1989).

The workshop's opening quiz activity involves participants in making computational estimates with money, whole numbers, fractions, decimals, and percentages. The problems are similar to those in the majority of elementary mathematics textbooks. The opening activity points out that these types of problems are traditionally used for repeated practice of particular skills being taught or reviewed. Finding exact answers is usually the objective. But, by having participants use their own strategies to solve these traditional textbook problems, the opening activity points out that the participants (and the students) already know something about estimating.

Focusing on the process of arriving at an estimate, rather than focusing on the answer, models for participants the importance of recognizing and discussing various strategies and estimates for the same problem.

Estimation strategies will be a new topic for many workshop participants. In a single workshop, it is difficult to bring participants from the introduction stage to the implementation stage. For participants to comfortably use estimation activities with their Chapter 1 students, they must personalize and internalize the workshop content to their teaching situation. Thus, it is important for the presenter to promote discussion of estimation as it relates to the participants' specific circumstances.

The presenter should also encourage participants to orally share the strategies they use. Although perhaps new to mathematics, talking about one's thinking is important not only to instruction in estimation, but to good instruction in general.

In the workshop, participants will take typical pages from a text or workbook and plan estination lessons using familiar materials. The implementation of estimation as a more advanced skill in Chapter 1 classrooms should not be considered as a separate unit or week-long topic. Rather, estimation activities should become part of every mathematics lesson and should replace some of the current drill and practice.

Research has found that spending less time on drill and practice and replacing it with estimation, mental computation, and problem-solving activities builds understanding of mathematical concepts and does not adversely affect computational skills. That "machines can make calculations, (but) mathematically literate people are needed to decide what kind of calculations should be made" is the attitude promoted by this workshop (Levenson, 1989).

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